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Band-spin–orbit-energy effects in conductivity of two-dimensional weakly disordered semiconductor systems

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Abstract. Effects of conduction-band splitting caused by the spin–orbit interaction on the magnetoresistance in strict two-dimensional disordered semiconductor systems are considered. The Dyakonov–Perel' process (the randomization of spin precession due to elastic scattering) is assumed to be the dominant spin-dephasing mechanism. Unlike results published previously, which turned out to be wrong, new expressions for the weak-localization corrections to the conductivity under an applied magnetic field are derived. The field dependence of the conductivity is proved to be of variable sign like that of metal films. The explicit forms of the correction are, however, markedly different from those known for the impurity spin–orbit scattering. Both cases—the perpendicular and parallel magnetic field, where an account of Zeeman splitting is necessary—are discussed.

1. Introduction

The problem of the current-carrier localization in disordered metals and semiconductors containing randomly spread impurities has been extensively studied during the past decade [1]. Accordance between theory and a large body of experimental results was most impressive in the weakly localized regime (for $\epsilon_F\tau \gg 1$, ϵ_F being the Fermi energy and τ the elastic lifetime), where a perturbational treatment from the metallic limit is applicable. In this regime, by moving an electron through a sample, the interference between time-reversal paths plays a particular role giving rise to various weak-localization effects. A phenomenon studied very frequently is a particular sensitivity of the electrical conductivity to variation in a magnetic field on a scale much smaller than that which determines the classical magnetoresistance. It has beeen shown both theoretically and experimentally that the sensitivity is controlled by such relaxation processes as inelastic scattering as well as scattering by paramagnetic and spin–orbit (SO) impurities [1]. Thus the observation of the magnetoresistance provides a method for determining the electron spin-flip time.

We will be interested, in this paper, in the case where the spin–orbit (SO) interaction is the main reason for spin-flip events. At low temperatures, the SO scattering in disordered metals is usually due to the electric field of heavy impurities. For the first time, the effect of SO impurities on the magnetoresistance was considered in [2]. Later on, this theory was complemented by taking into account the Zeeman energy [3]. Causes for spin relaxation in semiconductors are more numerous. Besides the spin–orbit interaction of degenerate valence bands in p-type materials and the Elliott mechanism of impurity relaxation [4], an important source of the relaxation is the intracrystalline electric field existing in non-centrosymmetric crystals of heteropolar semiconductor compounds. Because of the field, the

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conduction-electron Hamiltonian in bulk crystals includes an SO term linear in momentum (the p^1 term)[5]

$$H_{so}^{(1)} = \frac{\alpha}{\hbar}(p \times c) \cdot \sigma \tag{1.1}$$

in the case of A_2B_6 compounds with the polar space group C_{6v}^4, or cubic in momentum (the p^3 term)[6]

$$H_{so}^{(3)} = \gamma \left[\sigma_x p_x (p_y^2 - p_z^2) + \sigma_y p_y (p_z^2 - p_x^2) + \sigma_z p_z (p_x^2 - p_y^2)\right] \tag{1.2}$$

in the case of A_3B_5 compounds with the space group T_d^2. Here c is the unit vector along the polar axis, p is the electron momentum, σ is the Pauli spin-matrix vector, and α and γ are SO constants. Moreover the study of the spin-flip Raman scattering [7] and the optical orientation of electron spins [8] have produced strong evidence that the spin relaxation in the semiconductors is usually not due to heavy impurities, but is caused by the underlying band structure.

There are many similarities but there are also important differences between the impurity SO relaxation in metals and the relaxation due to the band SO coupling in semiconductors. In the former mechanism, the spin-flip event can happen only as a result of a collision with the SO impurity, hence the spin of an electron is conserved between the two successive collisions. One can choose a direction as the spin quantization axis. Then the spin relaxation of the electron is a stochastic process of transitions between two possible states: up and down with respect to this direction. It is important that we may allow the spin quantization axes of all electrons of a sample to be equally directed. In other words, one can choose a common quantization axis. Unlike this, the spin dephasing in the band SO coupling takes place through the Dyakonov–Perel' (DP) mechanism [9]: the band SO energy can be considered as the Zeeman energy in a fictitious magnetic field $B_f(p) = (\alpha/\hbar g \mu_B)(p \times c)$ which stochastically changes its direction by elastic scattering, giving rise to the spin relaxation. Between the collisions, for an electron of momentum p, the projection of its spin on the $B_f(p)$ direction is conserved. The common spin quantization axis is, however, absent—any projection of the total spin operator is no longer conserved. There is also another feature of the spin kinetics specific for systems whose electron Hamiltonian includes SO terms linear in momentum. It can be interpreted as the spin precession induced by diffusion [10]. The differences pointed out force us to assume that the two mechanisms should affect magnetotransport differently.

Up to now, little work has been done on incorporating the conduction-band SO coupling into the weak-localization theory of strict two-dimensional (2D) systems. A previous approach to the problem presented in [11] seems to be oversimplified. By considering 2D systems, they suggested a form of the Cooperon propagator like that known for the impurity SO scattering. No evidence of the validity of such an assumption has been produced. Regrettably, the important consequences of the lack of central symmetry have escaped their attention, and the final results turned out to be wrong.

The purpose of this paper is to develop the weak-localization theory for purely 2D systems where the DP mechanism plays the dominant role. Conduction-band splitting in heterostructures (and quantum wells) produced from II–VI compounds in such a way that the electron-motion plane is perpendicular to the polar vector c is to be described by the Hamiltonian $H_{so}^{(1)}$. There are weighty arguments, including the interface electric field and the strain induced by the dispersive forces, for the presence of the p^1 term also in the electron Hamiltonian of III–V heterostructures. The p^3 term also can be responsible for spin splitting. At low electron density, when only the ground state of a confining potential is occupied, the contribution of the term to the effective 2D Hamiltonian can be obtained

by obtaining the expectation value of $H^{(3)}$ for the state. In this way, for an orientation of the structure along [001], one obtains

$$H_{so}^{(3.1)} = \frac{\beta}{\hbar}\sigma \cdot \kappa(p) \tag{1.3}$$

where $\kappa = (-p_x, p_y, 0)$ and $\beta = \gamma\langle p_z^2\rangle$. This expression holds only at small filling of the ground state, when $p_F^2 \ll \langle p_z^2\rangle$ (p_F being the Fermi momentum). At higher filling, terms cubic in momentum are also to be taken into account. In most of the present paper we assume the p^1 term is the main reason for the spin relaxation. So the theory should be applicable, in the first place, to A_2B_6 heterostructures. There are also experimental indications that the p^1 term may dominate in some narrow-band-gap A_3B_5 heterostructures [12]. The opposite limit, when the p^1 term is negligible in comparison with the $H_{so}^{(3.1)}$ Hamiltonian, is briefly considered at the end. Corrections to the magnetoconductivity are shown to have the same form in both limiting cases. The results obtained differ quantitatively and, in some respects, qualitatively from those known for the impurity SO interaction, e.g. impurity SO coupling does not result in magnetoconductivity of variable sign in purely 2D systems, whereas the band SO interaction does. Thus the conductivity of a purely 2D system with the band SO coupling turns out to be similar in form to that of a metal film [2, 3].

The cases when the Hamiltonians $H_{so}^{(1)}$ and $H_{so}^{(3.1)}$ make comparable contributions to the spin relaxation and when, in addition to the band SO coupling, the impurity SO scattering should be taken into account, will not be touched on. However, the methods developed in the present paper will undoubtedly be useful in the general situation.

It should be noted that inapplicability of the theory [2] to strict 2D systems was mentioned even in the first works [13,14] in which antilocalization in 2D semiconductor structures was observed. An attempt to interpret experimental results with the help of this theory, made in the recent paper [15], seems to be incorrect. By fitting the magnetoconductance data with a formula for the quantum correction to the conductance derived in [2], they assumed $2H_{so}^x = H_{so}$ and $H_{so}^z = 0$ (in the notations of [2] and [15]). At the impurity spin-relaxation mechanism, this assumption supposes the possibility for the electron momentum perpendicular to the system plane to change during the scattering process, which is (under the confinement conditions) at variance with quantum mechanics. If one adopts the relaxation mechanism due to the band SO coupling, formulas of [2], as will be shown below, cease to be applicable. Therefore, the expression for the conductance used in [15] does not apparently correspond to any physical system.

The outline of this paper is as follows. In section 2, a description of the model is given and the general spinor structure of the Cooperon propagator is discussed. The well known difficulty of dealing with SO coupling relates to a non-trivial spin dependence acquired by scattering amplitudes, vertices and other attributes of the theory. The analogous obstacles in the diffuson propagator problem have been recently removed with the help of a novel form of tensor products of the Pauli matrices [10]. In order to take full advantage of a technique of [10], we derive a relationship between the electron Green function and its time-reversal counterpart and then, with the help of the relation, make a map from the Cooperon problem to the quasidiffuson one. In section 3, a systematic scheme for investigating the Bethe–Salpeter (transport) equation for the Cooperon propagator is presented. We find eigenfunctions of the kernel of the equation and obtain an explicit form of the Cooperon propagator in the absence of an external magnetic field. In section 4, diamagnetic effects are studied. By means of the transport equation written in the coordinate space, where it looks like an equation describing the diffusion accompanied by the spin precession, an expression for the magnetoconductivity in the perpendicular magnetic field is obtained. In

section 5, we take into account the Zeeman energy and find the magnetoconductivity when the magnetic field B is applied in the plane of the 2D electron gas. Comments on systems with the $H_{so}^{(3.1)}$ Hamiltonians are given in section 6. We give in section 7 a summary of our results and conclusions.

It will be assumed below that the reader is familiar with the many-body quantum-field theory (the Matsubara method, the Feynman diagram technique, etc). For a review on these topics we refer to the textbook [16]. A familiarity with [10] is advised as well.

2. The general formula for the conductivity

We consider the degenerate two-dimensional Fermi gas of independent electrons of charge e, spin $\frac{1}{2}$, and magnetic moment $\mu = \frac{1}{2}g\mu_B\sigma$ scattered by normal impurities and subjected to an external magnetic field B. Then the Hamiltonian is written as

$$H = H_0\left(p - \frac{e}{c}A\right) + H_Z + H_{imp} \tag{2.1}$$

where

$$H_0(p) = \frac{p^2}{2m} + \alpha(p \times c)\cdot\sigma \tag{2.2}$$

$$H_Z = -\mu\cdot B \tag{2.3}$$

$$H_{imp} = \sum_i U\delta(r - R_i). \tag{2.4}$$

Here A is the vector potential, r is the position of an electron, R_i are the positions of the arbitrarlly distributed short-range impurities of concentration n_{imp}. In equations (2.1)–(2.3) and below we set $\hbar = 1$. A standard technique [16] tells us that the elastic lifetime τ is given by

$$\tau^{-1} = mn_{imp}U^2. \tag{2.5}$$

We shall consider the system under the following conditions.

(1) The SO energy, αp_F, is small so that the parameter $\eta = 2\alpha p_F\tau$ controlling the kinetics of spin-flip processes is small

$$\eta \ll 1. \tag{2.6}$$

(2) The magnetic field is also small

$$\omega_c\tau \ll 1 \qquad \omega_s\tau_{so} \ll 1 \tag{2.7}$$

where

$$\omega_c = \frac{|e|B}{mc} \qquad \omega_s = |g|\mu_B B \tag{2.8}$$

are the cyclotron and paramagnetic resonance frequencies, and

$$\tau_{so}^{-1} = \tau^{-1}\eta^2 \tag{2.9}$$

is the spin relaxation time [9]. More precisely, equation (2.9) gives the order of magnitude of the inverse spin scattering rate. The longitudinal and transverse relaxation times, T_1 and T_2, differ from (2.9) by numerical factors of the order of unity.

Let us first consider the system neglecting the diamagnetic effects. As known [1], the weak-localization correction comes from the so-called Cooperon $(\alpha\gamma|C(\omega, q, B)|\rho\beta)$

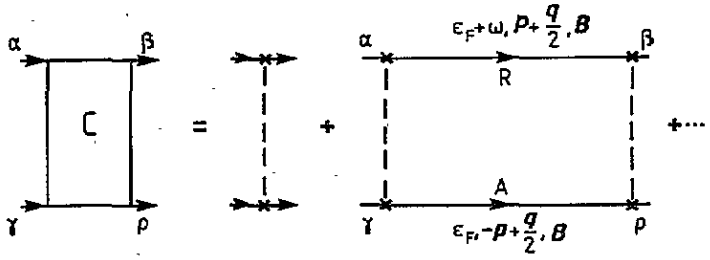

Figure 1. Ladder graphs for the Cooperon.

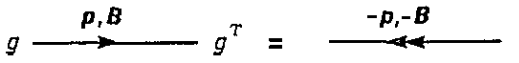

Figure 2. The graphical representation of the arrow-reversal operation. The double arrow stands for the opposite sign of the magnetic field.

shown in figure 1, where the solid line represents the impurity-averaged one-particle Green function

$$G^{R(A)}(\zeta, p, B) = \left[\zeta - H_0(p) + \mu \cdot B \pm \frac{i}{2\tau}\right]^{-1} \tag{2.10}$$

and the dashed line between two crosses corresponds to the factor $(m\tau)^{-1}$. Here the superscript R (A) stands for the retarded (advanced) part of the function. For the reasons mentioned in the introduction, it is convenient to the reverse direction of one of two electron lines forming the Cooperon ladder. This transformation, which has meaning of time reversal, can be carried out by means of the equality

$$g_{\kappa\zeta} G_{\zeta\xi}(\epsilon, p, B) g^{T}_{\xi\nu} = G_{\nu\kappa}(\epsilon, -p, -B) \tag{2.11}$$

easily verified using the identity

$$\mathbf{g} \cdot \boldsymbol{\sigma}^{T} \cdot \mathbf{g}^{T} = -\boldsymbol{\sigma} \tag{2.12}$$

where $\mathbf{g} = i\boldsymbol{\sigma}_2$, the superscript of T denotes transposition and the summation convention for repeated indices is used. In the diagram language, the operation of the arrow reversal is exhibited in figure 2. By applying (2.11) to the lower electron propagators of each graph of the Cooperon ladder, one can express, as shown in figures 3 and 4, the Cooperon in terms of an amplitude, $(\alpha\gamma|K(\omega, q, B)|\nu\rho)$,

$$(\alpha\gamma|C|\rho\beta) = g^{T}_{\gamma\kappa}(\alpha\kappa|K|\nu\beta) g_{\nu\rho} \tag{2.13}$$

which is a sum of diffuson-type ladder diagrams. The short-range character of the impurity potential enables one, in much the same way as in [10], to perform the integration over internal momenta of each graph of the quasidiffuson ladder and to present the Bethe–Salpeter (transport) equation in the form of a linear matrix equation

$$(\gamma\beta|K(\omega, q, B)|\alpha\delta) = \frac{1}{m\tau}\delta_{\gamma\delta}\delta_{\alpha\beta} + (\gamma\beta|T(\omega, q, B)|\kappa\zeta)(\kappa\zeta|K(\omega, q, B)|\alpha\delta) \tag{2.14}$$

where the kernel function $T(\omega, q, B)$ is given by

$$(\gamma\beta|T(\omega, q, B)|\alpha\lambda) = \frac{1}{m\tau}\int \frac{d^2p}{(2\pi)^2} G^{R}_{\gamma\lambda}(\epsilon_F + \omega, p + q/2, B) G^{A}_{\alpha\beta}(\epsilon_F, p - q/2, -B). \tag{2.15}$$

The kernel $T(\omega, q, B)$ coincides with that for the diffuson problem of [10] except for the magnetic-field dependence. Owing to the inequalities (2.6) and (2.7) and at small values of wave vectors

$$ql \ll 1 \tag{2.16}$$

where $l = v_F\tau$ is the mean free path and $v_F = p_F/m$ is the Fermi velocity, it is sufficient to evaluate the kernel to the first order in $\omega_s\tau$ and to the second order in ql

$$T(\omega, q, B) \simeq T_0(\omega) + T_1(B) + T_1(q) + T_2(q). \tag{2.17}$$

The kernels $T_0(\omega)$ and $T_{1,2}(q)$ were calculated in [10]. $T_1(B)$ can be calculated in the same way. As a result we have

$$(\gamma\beta|T_0(\omega)|\alpha\lambda) = \frac{w}{2}\delta_{\gamma\beta}\delta_{\alpha\lambda} + \frac{u}{2}(c\cdot\sigma)_{\gamma\beta}(c\cdot\sigma)_{\alpha\lambda} + \frac{v}{2}(c\times\sigma)^n_{\gamma\beta}(c\times\sigma)^n_{\alpha\lambda} \tag{2.18}$$

$$(\gamma\beta|T_1(B)|\alpha\lambda) = -\frac{i}{2}\frac{g}{|g|}\Omega\left[(h\cdot\sigma)_{\gamma\beta}\delta_{\alpha\lambda} + \delta_{\gamma\beta}(h\cdot\sigma)_{\alpha\lambda}\right] \tag{2.19}$$

$$(\gamma\beta|T_1(q)|\alpha\lambda) = \tfrac{1}{2}i\eta Q e_{ijk}(\hat{q}\times c)^i\sigma^j_{\gamma\beta}\sigma^k_{\alpha\lambda} \tag{2.20}$$

$$(\gamma\beta|T_2(q)|\alpha\lambda) = -\tfrac{1}{4}Q^2\left[\delta_{\gamma\beta}\delta_{\alpha\lambda} + (c\cdot\sigma)_{\gamma\beta}(c\cdot\sigma)_{\alpha\lambda} + (c\times\sigma)^i_{\gamma\beta}(c\times\sigma)^i_{\alpha\lambda}\right] \tag{2.21}$$

where

$$w \approx 1 + i\omega\tau \qquad u \approx 1 + i\omega\tau - \eta^2 \qquad v = \frac{u+v}{2} \tag{2.22}$$

$$h = B/B \qquad \Omega = \omega_s\tau \qquad Q = ql. \tag{2.23}$$

The indices i, j, k, n refer to the 3D-vector Cartesian components (with the summation convention) and e_{ijk} is the 3D antisymmetric tensor. In writing equations (2.19)–(2.21), we retained only leading terms in the parameter η. The appearance of the kernel $T_1(q)$ linear in wave vector is the essential ingredient of the theory. One should emphasize the important feature of the expressions (2.18)–(2.24): all of them have a separable form—every term is the tensor product of a matrix which depends only on the indices (γ, β) entering the kernel T from the left (see figure 4) and a matrix which depends only on the right-side indices (α, λ). This arrangement of the spinor indices appears to be possible owing to identities for the tensor products of the Pauli matrices presented in appendix A of [10]. The free term of the matrix equation (2.14), in accordance with (A.1.1) of [10],

$$\delta_{\gamma\delta}\delta_{\alpha\beta} = \tfrac{1}{2}[\delta_{\gamma\beta}\delta_{\alpha\delta} + (c\cdot\sigma)_{\gamma\beta}(c\cdot\sigma)_{\alpha\delta} + (c\times\sigma)^n_{\gamma\beta}(c\times\sigma)^n_{\alpha\delta}] \tag{2.24}$$

can be transformed to such a form as well. It is clear, therefore, that the solution of equation (2.14) has to be of the same form

$$(\gamma\beta|K(\omega, q, B)|\alpha\delta) = \sum_{i,j=0}^{3}\chi^i_{\gamma\beta}K_{i,j}\chi^i_{\alpha\delta} \tag{2.25}$$

where

$$\chi^{(0)}_{\alpha\kappa} = \delta_{\alpha\kappa} \qquad \chi^{(1,2)}_{\alpha\kappa} = (c\times\sigma)^{(1,2)}_{\alpha\kappa} \qquad \chi^{(3)}_{\alpha\kappa} = (c\cdot\sigma)_{\alpha\kappa}. \tag{2.26}$$

The graphical representation of (2.25) is given in figure 5.

Now we turn to the contribution which the Cooperon makes to the conductivity. The simplest diagram containing the Cooperon is schematically given in figure 6(a). Note that the velocity operator of the system

$$v(p) = i[H_0(p), r] = \frac{p}{m} + \alpha(c\times\sigma) \tag{2.27}$$

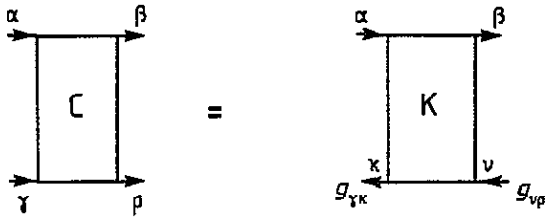

Figure 3. The relationship between the Cooperon C and the quasidiffuson K.

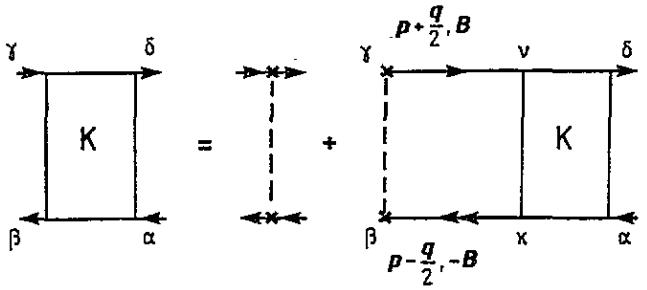

Figure 4. The Bethe–Salpeter equation for the quasidiffuson.

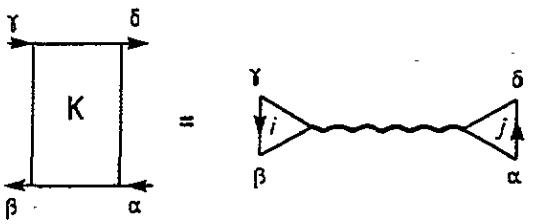

Figure 5. The spinor structure of the quasidiffuson. The left and right triangles correspond to the matrices χ. The arrows show the order of spinor indices.

as well as the usual scalar part also has a spin component. Now we perform a series of simple transformations of the graph 6(a). (i) Let us, in accordance with (2.13), substitute the quasidiffuson for the Cooperon and (ii) apply the time-reversal operation (2.11) to two Green functions, which join the Cooperon with the upper velocity vertex $v(-p)$ and, using the equality

$$\mathbf{g} \cdot v^{\mathrm{T}}(-p) \cdot \mathbf{g}^{\mathrm{T}} = -v(p) \tag{2.28}$$

to this vertex. It is easy to see that as a result of these manipulations both the $\mathbf{g}$ matrices, which after perfoming the prescription (i) were on opposite sides of the quasidiffuson (see figure 3), go to its right-hand side. (iii) Now one should utilize the representation (2.25) and (iv) introduce the time-reversal set of spinor functions

$$\chi^{\mathrm{t(j)}} = \mathbf{g} \cdot \chi^{\mathrm{T(j)}} \cdot \mathbf{g}^{\mathrm{T}} = [1, -(c \times \sigma), -(c \cdot \sigma)]\,. \tag{2.29}$$

After all these transformations are performed the graph 6(a) acquires the form shown in figure 6(b) in which the quasidiffuson looks like a Bose-type excitation carrying out an interaction between the electrons. Spinor indices of every entity of the graph are arranged clockwise. All other possible one-Cooperon diagrams can be generated from this by making the velocity-vertex renormalization due to impurity scattering as well as by inserting a single-impurity line so that it intersects the quasidiffuson line is shown in figure 7. A straightforward investigation shows that because of the inequalities (2.6) and (2.7) these complications result in negligible corrections to the value of the graph 6(a) and can be ignored. Moreover, for the same reasons one can set the field B, the wave vector q, and

the SO constant α equal to zero in all parts of the diagram except for the quasidiffuson. So the quantum correction to the conductivity takes the form

$$\delta\sigma = 2\sigma_0\tau^2 \int_{|q|\leqslant q_c} \frac{d^2q}{(2\pi)^2}(K_{00} - K_{11} - K_{22} - K_{33}) \tag{2.30}$$

where $\sigma_0 = e^2\epsilon_F\tau/\pi$ is the Drude conductivity, $q_c = (D\tau)^{-1/2}$ is the cut-off momentum, and $D = v_F^2\tau/2$ is the diffusion constant.

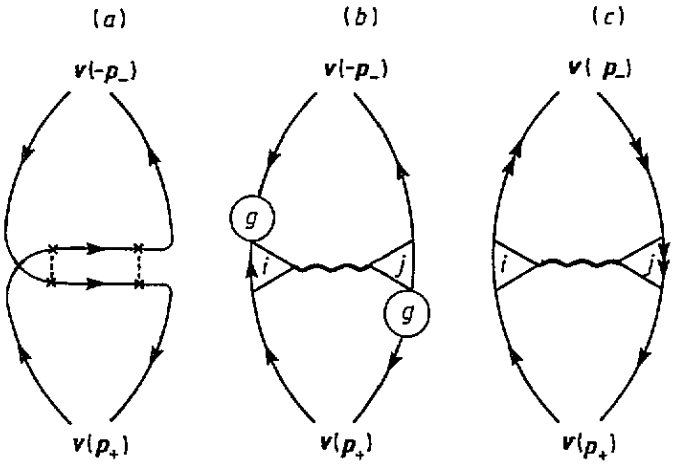

Figure 6. Transformations of the simplest graph contributing to the weak-localization correction. (a) The initial form ($p_\pm = p \pm q/2$). (b) The same graph after the substitution of equations (2.11) and (2.25) (wiggly line is the quasidiffuson). (c) The final form obtained after transferring the left-hand $\mathbf{g}$ matrix from χ_i through the upper part of the graph to χ_j. Here the double arrow on the right triangle (with the label j) denotes the time-reversed spinor vector matrix $\chi_j^\dagger = \mathbf{g}^T \cdot \chi_j^T \cdot \mathbf{g} = \{\delta, -(c \times \sigma), -(c \cdot \sigma)\}$.

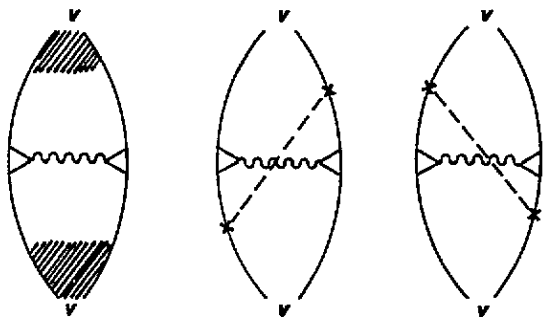

Figure 7. The diagrams contributing to the quantum correction to the conductivity to first order in $(\epsilon_F\tau)^{-1}$. Hatching symbolizes the impurity renormalization.

3. Spectral representation for the quasidiffuson

In this section, we assume that the magnetic field is absent. As usual, we replace ω in the kernel T of the transport equation (2.14) by the inverse of the energy relaxation time τ_ϵ.

$$-i\omega\tau \longrightarrow \frac{\tau}{\tau_\epsilon} = \phi. \tag{3.1}$$

Then the kernel takes the form

$$(\alpha\beta|T(q)|\gamma\delta) = \tfrac{1}{2}a(q)\delta_{\alpha\beta}\delta_{\gamma\delta} + \tfrac{1}{2}b(q)(c\cdot\sigma)_{\alpha\beta}(c\cdot\sigma)_{\gamma\delta} + \tfrac{1}{2}d(q)(c\times\sigma)^n_{\alpha\beta}(c\times\sigma)^n_{\gamma\delta} + \frac{i}{2}\eta Q(\hat{q}\times c)^n e_{njk}\sigma^j_{\alpha\beta}\sigma^k_{\gamma\delta} \tag{3.2}$$

where

$$1-a(q)\approx\phi+\frac{Q^2}{2} \qquad 1-b(q)\approx\phi+\eta^2+\frac{Q^2}{2} \qquad 1-d(q)\approx\phi+\frac{\eta^2}{2}+\frac{Q^2}{2}. \tag{3.3}$$

The solution of the equation (2.14) is obtained by inverting the operator $1-T$. In order to do this, as is well known, one needs to solve the eigenvalue problem for the kernel T, i.e. to find a complete set of orthogonal eigenvectors $\Phi^{(j)}$ such that

$$(\alpha\beta|T|\gamma\delta)(\delta\gamma|\Phi^{(i)}) = \epsilon_{(i)}(\alpha\beta|\Phi^{(i)}). \tag{3.4}$$

Summation over repeated indices is implicit throughout. An eigenvector $\Phi^{(i)}$ is a 2×2 matrix normalized 'to 2'

$$(\Phi^{(i)}|\Phi^{(j)}) = (\Phi^{*(j)}|\alpha\beta)(\beta\alpha|\Phi^{(j)}) = 2\delta_{ij} \tag{3.5}$$

*where the sign * means that if*

$$(\alpha\beta|\Phi) = \sum_{n=0}^{3} c_n\gamma^{(n)}_{\alpha\beta} \qquad \gamma^n = (1,\sigma_x,\sigma_y,\sigma_z) \tag{3.6}$$

then

$$(\Phi^*|\beta\alpha) = \sum_{n=0}^{3} c_n^*\gamma^{(n)}_{\beta\alpha} \tag{3.7}$$

where c_n^* is the complex conjugate of c_n. There are four eigensolutions of equation (3.4). One evidently is

$$\epsilon_{(0)} = a(q) \qquad \Phi^{(0)} = 1. \tag{3.8}$$

The other three can be looked for in the form

$$\Phi = F\cdot(c\times\sigma) + S(c\cdot\sigma). \tag{3.9}$$

Substituting (3.9) in (3.4) and employing equations (2.16)–(2.22), one may easily derive a set of equations from which the 2D vector F and the scalar S may be determined. We write the result in the form

$$d(q)F - t(\hat{q}\times c)S = \epsilon F \tag{3.10}$$

$$tF\cdot(\hat{q}\times c) + b(q)S = \epsilon S \tag{3.11}$$

where $t = i\eta Q$. Solutions of the system have the form

$$\epsilon_{(1)} = \frac{b+d}{2} + \left[\left(\frac{b-d}{2}\right)^2 - t^2\right]^{1/2} \qquad \Phi^{(1)} = \frac{1}{N_{(1)}}\left(\frac{t}{\epsilon_{(1)}-b}c\cdot\sigma - \hat{q}\cdot\sigma\right) \tag{3.12}$$

$$\epsilon_{(2)} = \frac{b+d}{2} - \left[\left(\frac{b-d}{2}\right)^2 - t^2\right]^{1/2} \qquad \Phi^{(2)} = \frac{1}{N_{(2)}}\left(c\cdot\sigma + \frac{t}{\epsilon_2-d}\hat{q}\cdot\sigma\right) \tag{3.13}$$

$$\epsilon_3 = d \qquad \Phi^3 = \hat{q}\cdot(c\times\sigma) \tag{3.14}$$

where

$$N_{(1)} = \left(1 + \frac{\epsilon_{(1)} - d}{\epsilon_{(1)} - b}\right)^{1/2} \qquad N_{(2)} = \left(1 + \frac{\epsilon_{(2)} - b}{\epsilon_{(2)} - d}\right)^{1/2} \tag{3.15}$$

and the argument q of the functions $a(q), b(q)$, and $d(q)$ is omitted for short. Now it is not difficult to verify the validity of the spectral representation for the free term

$$\frac{1}{2m\tau}\left[\delta_{\alpha\beta}\delta_{\gamma\delta} + (c\cdot\sigma)_{\alpha\beta}(c\cdot\sigma)_{\gamma\delta} + (c\times\sigma)^n_{\alpha\beta}(c\times\sigma)^n_{\gamma\delta}\right] = \frac{1}{2m\tau}\sum_{i=0}^{3}(\alpha\beta|\Phi^{(i)})(\Phi^{*(i)}|\gamma\delta) \tag{3.16}$$

and for the kernel

$$(\alpha\beta|T|\gamma\delta) = \tfrac{1}{2}\sum_{i=0}^{3}\epsilon_{(i)}(\alpha\beta|\Phi^{(i)})(\Phi^{*(i)}|\gamma\delta). \tag{3.17}$$

Owing to (3.16) and (3.17), the solution of equation (2.14) can be written as

$$(\alpha\beta|K|\gamma\delta) = \frac{1}{2m\tau}\sum_{i=0}^{3}\frac{(\alpha\beta|\Phi^{(i)})(\Phi^{*(i)}|\gamma\delta)}{1-\epsilon_{(i)}}. \tag{3.18}$$

A more convenient form can be obtained if one utilizes the explicit form of the eigenvectors and eigenvalues

$$\begin{aligned}(\alpha\beta|K|\gamma\delta) = \frac{1}{2m\tau}\Big[&A\delta_{\alpha\beta}\delta_{\gamma\delta} + B(c\cdot\sigma)_{\alpha\beta}(c\cdot\sigma)_{\gamma\delta} + C_{ij}(c\times\sigma)^i_{\alpha\beta}(c\times\sigma)^j_{\gamma\delta}\\ &+M\cdot(c\times\sigma)_{\alpha\beta}(c\cdot\sigma)_{\gamma\delta} - (c\cdot\sigma)_{\alpha\beta}M\cdot(c\times\sigma)_{\gamma\delta}\Big]\end{aligned} \tag{3.19}$$

where the following abbreviations have been introduced:

$$A = (1-a)^{-1} \tag{3.20}$$

$$B = \frac{1-d}{(1-b)(1-d) - \eta^2 Q^2} \tag{3.21}$$

$$C^{ij} = \frac{\hat{q}^i\hat{q}^j}{1-d} + \frac{(1-b)(\hat{q}\times c)^i(\hat{q}\times c)^j}{(1-b)(1-d) - \eta^2 Q^2} \tag{3.22}$$

$$M = -\frac{\beta(\hat{q}\times c)}{(1-b)(1-d) - \eta^2 Q^2}. \tag{3.23}$$

It is important to note that the expressions (3.21)–(3.23) are not reducible to a sum of simple diffusion poles. An attempt at factoring the denominators, e.g. at $\phi \ll \eta^2$

$$(1-b)(1-d) - \eta^2 Q^2 = \tau^2\left[Dq^2 - \frac{1}{2\tau_{so}}(1+i\sqrt{7})\right]\left[Dq^2 - \frac{1}{2\tau_{so}}(1-i\sqrt{7})\right] \tag{3.24}$$

results in the appearance of complex 'relaxation times', whose physical meaning is not clear. Substituting (3.19)–(3.23) in (2.30), we find at $\tau_{so} \ll \tau_\epsilon$

$$\frac{\delta\sigma(0)}{\sigma_0} = \frac{1}{4\pi\epsilon_F\tau}\left[\ln\frac{\tau}{\tau_\epsilon} - 3\ln\frac{\tau}{\tau_{so}} - C_1\right] \tag{3.25}$$

where

$$C_1 = 2\ln 2 + \frac{8}{\sqrt{7}}\left(\frac{\pi}{2} + \tan^{-1}\frac{1}{\sqrt{7}}\right) \approx 7. \tag{3.26}$$

4. Magnetoconductivity in a perpendicular field

It is known [3] that when an external magnetic field is applied perpendicular to the plane of electron motion, the an effects of Zeeman splitting are much smaller than that of diamagnetism and can be dropped. In small fields the diamagnetic effects can be taken into account within the semiclassical phase-integral approximation for the Green function [17]

$$G(r_1, r_2, \zeta, B) = G(r_1 - r_2, \zeta) \exp\left(\frac{ie}{c}\int_{r_1}^{r_2} A(s)\,ds\right) \tag{4.1}$$

where $A(r)$ is the vector potential and the line integral is performed along a straight line between r_1 and r_2. By utilizing (4.1) and methods of [18], one can rewrite the transport equation (2.14) in the coordinate space

$$(\alpha\gamma|K(r, r')|\rho\beta) = \frac{\delta(r - r')}{2\pi\tau}\left[\delta_{\alpha\gamma}\delta_{\rho\beta} + (c\cdot\sigma)_{\alpha\gamma}(c\cdot\sigma)_{\rho\beta} + (c\times\sigma)^n_{\alpha\gamma}(c\times\sigma)^n_{\rho\beta}\right]$$
$$+(\alpha\gamma|T(\Pi)|\lambda\kappa)(\kappa\lambda|K(r, r')|\rho\beta) \tag{4.2}$$

where the kernel $T(\Pi)$,

$$(\alpha\gamma|T(\Pi)|\rho\beta) = \tfrac{1}{2}\left[a(\Pi)\delta_{\alpha\gamma}\delta_{\rho\beta} + b(\Pi)(c\cdot\sigma)_{\alpha\gamma}(c\cdot\sigma)_{\rho\beta} + d(\Pi)(c\times\sigma)^n_{\alpha\gamma}(c\times\sigma)^n_{\rho\beta}\right]$$
$$+\frac{i}{2}\eta l(\Pi\times c)^n e_{njk}\sigma^j_{\alpha\gamma}\sigma^k_{\rho\beta} \tag{4.3}$$

is obtained from the kernel $T(q)$ of equation (3.2) by the substitution of the differential operator Π,

$$\Pi = \frac{\nabla}{i} + \frac{2e}{c}A(r) \tag{4.4}$$

for the wave vector q.

In the presence of the magnetic field, it is convenient to deal with the spinor basis $\{f_n\}$,

$$f_n = (f_{(0)}, f_{(1)}, f_{(2)}, f_{(3)}) = (1, \sigma_+, \sigma_-, \sigma_z) \qquad \sigma_\pm = \frac{1}{\sqrt{2}}(\sigma_x \pm i\sigma_y) \tag{4.5}$$

instead of the basis $\{\gamma_n\}$ of (3.6). Let us also introduce creation and annihilation operators

$$b = \frac{\lambda}{2}(\Pi_x - i\Pi_y) \qquad b^+ = \frac{\lambda}{2}(\Pi_x + i\Pi_y) \qquad \lambda^2 = \frac{c}{eB} \tag{4.6}$$

obeying the Bose-type commutation relation. Then the kernel $T(\Pi)$ takes the form

$$(\alpha\gamma|T(\Pi)|\rho\beta) = \sum_{i,j=0}^{3}(T^{(0)}_{ij} + T^{(1)}_{ij}) f_{(i)\alpha\gamma} f^*_{(j)\rho\beta} \tag{4.7}$$

where

$$\mathbf{T}^{(0)} = \tfrac{1}{2}\begin{pmatrix} a(\Pi) & 0 & 0 & 0 \\ 0 & d(\Pi) & 0 & 0 \\ 0 & 0 & d(\Pi) & 0 \\ 0 & 0 & 0 & b(\Pi) \end{pmatrix} \tag{4.8}$$

$$\mathbf{T}^{(1)} = \frac{i}{2}q_{so}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \\ 0 & 0 & 0 & b^+ \\ 0 & -b^+ & -b & 0 \end{pmatrix} \tag{4.9}$$

$$q_{so} = \eta \frac{l}{\lambda}\sqrt{2} \tag{4.10}$$

the $\hat{z}$ axis is assumed to be directed along c and one should substitute $(4/\lambda^2)(b^+b + \frac{1}{2})$ instead of Π^2 in the functions a, b, and d entering the matrix $T^{(0)}$. Again, just as in the preceding section, the equation (4.2) can be solved once the eigenvalues and eigenfunctions of $T(\Pi)$ are found

$$(\alpha\gamma|T(\Pi)|\rho\beta)(\beta\rho|\Phi^{(i)}(r)) = \epsilon^i(\alpha\gamma|\Phi^{(i)}(r)). \tag{4.11}$$

The eigenfunctions can be formed from the Landau states $\psi_{m,k}(r) = \langle r|m, k\rangle$ for a doubly charged particle moving in the uniform magnetic field (where m is the main quantum number and k is a quantum number related to the degeneracy of the Landau levels). Let us define some symbols by

$$\begin{aligned} 1 - a_m &= \phi + \left(\frac{l}{\lambda}\right)(2m+1) \\ 1 - b_m &= \phi + \eta^2 + \left(\frac{l}{\lambda}\right)(2m+1) \\ 1 - d_m &= \phi + \frac{\eta^2}{2} + \left(\frac{l}{\lambda}\right)(2m+1). \end{aligned} \tag{4.12}$$

One subset of the eigenvectors is

$$\Phi^{(sc)}(m) = f_{(0)}|m\rangle \qquad \epsilon_{(sc)}(m) = a_m \qquad m = 0,\ 1,\ \ldots. \tag{4.13}$$

Here the coordinate dependence of the ket vector $|m\rangle$ is implicit as well as the dependence on the degeneracy quantum number. The other subsets of solutions can be looked for in the form

$$\Phi^{(i)}(m) = X_1(m)f_1|m-1\rangle + X_2(m)f_2|m+1\rangle + X_3(m)f_3|m\rangle. \tag{4.14}$$

Then (4.14) transforms into the linear-equation system

$$d_{m-1}X_1(m) + iq_{so}\sqrt{m}X_3(m) = \epsilon(m)X_1(m) \tag{4.15}$$

$$d_{m+1}X_2(m) + iq_{so}\sqrt{m+1}X_3(m) = \epsilon(m)X_2(m) \tag{4.16}$$

$$-i(\sqrt{m}X_1(m) + \sqrt{m+1}X_2(m)) + b_mX_3 = \epsilon(m)X_3(m). \tag{4.17}$$

Among the solutions of (4.15)–(4.17), there are three particular ones. The first corresponds to $m = -1$

$$\Phi^{(sp)}(-1) = f_{(2)}|0\rangle \qquad \epsilon_{(sp)}(-1) = d_0 \tag{4.18}$$

and the other two correspond to $m = 0$

$$\Phi_+^{(sp)}(0) = \frac{1}{N_{(+)}}\left[f_{(2)}|1\rangle - \frac{iq_{so}}{\epsilon_{(sp)}^{(+)} - b_0}f_{(3)}|0\rangle\right] \qquad \epsilon_{(sp)}^{(+)} = \frac{d_1+b_0}{2} + \sqrt{\left(\frac{d_1-b_0}{2}\right)^2 + q_{so}^2} \tag{4.19}$$

$$\Phi_-^{(sp)}(0) = \frac{1}{N_{(-)}}\left[\frac{iq_{so}}{\epsilon_{(sp)}^{(-)} - d_1}f_{(2)}|1\rangle + f_3|0\rangle\right] \qquad \epsilon_{(sp)}^{(-)} = \frac{d_1+b_0}{2} - \sqrt{\left(\frac{d_1-b_0}{2}\right)^2 + q_{so}^2} \tag{4.20}$$

where

$$N_{(+)} = \left(1 + \frac{\epsilon_{(+)} - d_1}{\epsilon_{(+)} - b_0}\right)^{1/2} \qquad N_{(-)} = \left(1 + \frac{\epsilon_{(-)} - b_0}{\epsilon_{(-)} - d_1}\right)^{1/2}. \tag{4.21}$$

At $m \geqslant 1$, when all three components of the eigenvector $\Phi^{(i)}$ (X_1, X_2, and X_3) are non-zero let us introduce more quantities

$$H_{\rm tr} = \frac{c}{4eD\tau} \qquad H_\epsilon = \frac{c}{4eD\tau_\epsilon} \qquad H_{\rm so} = \frac{c}{4eD\tau_{\rm so}} \tag{4.22}$$

$$A_m = m + \tfrac{1}{2} + \frac{H_\epsilon}{B} \tag{4.23}$$

$$B_m = m + \tfrac{1}{2} + \frac{H_\epsilon}{B} + \frac{H_{\rm so}}{B} \tag{4.24}$$

$$D_m = m + \tfrac{1}{2} + \frac{H_\epsilon}{B} + \frac{H_{\rm so}}{2B} \tag{4.25}$$

$$C_m = \left(m\frac{H_{\rm so}}{B}\right)^{1/2} \tag{4.26}$$

and rewrite the equations (4.15)–(4.17) in the form

$$H_{ij}(m)X_j^{(\rm sp)}(m) = \xi_{(\rm sp)}(m)X_i^{(\rm sp)}(m) \tag{4.27}$$

where

$$\xi(m) = [1 - \epsilon(m)]\frac{H_{\rm tr}}{B} \tag{4.28}$$

$$H_{ij}(m) = \begin{pmatrix} D_{m-1} & 0 & -{\rm i}C_m \\ 0 & D_{m+1} & -{\rm i}C_{m+1} \\ {\rm i}C_m & {\rm i}C_{m+1} & B_m \end{pmatrix}. \tag{4.29}$$

For every $m \geqslant 1$, the system (4.27) has three solutions (labelled by index s, $s = 1, 2, 3$). The explicit form of the solutions is cumbersome as it is determined by roots of the characteristic equation, $\det(\xi - H(m)) = 0$. Fortunately, there is no necessity to have the explicit form because the Cooperon contribution to the conductivity is determined only by the eigenvalues until the diffusion approximation is valid [2] and the eigenvalues enter the conductivity, as it will be seen, via the trace of the matrix $\mathbf{H}^{-1}(m)$. Following [2], one should change the integration in (2.30) by the summation on the eigenvalues $\epsilon^{(i)}$ of the operator $T(\Pi)$

$$\int \frac{{\rm d}^2q}{(2\pi)^2} \to \frac{eB}{\pi c}\sum_{\epsilon^{(i)}}. \tag{4.30}$$

Then (2.30) acquires the form

$$\frac{\delta\sigma(B)}{\sigma_0} = 2\frac{eB\tau}{m\pi c}\left\{\sum_{m=0}^{M}\frac{1}{1-\epsilon_{(\rm sc)}(m)} - \frac{1}{1-\epsilon_{(\rm sp)}(-1)} - \frac{1}{1-\epsilon_{(\rm sp)}^{(+)}} - \frac{1}{1-\epsilon_{(\rm sp)}^{(-)}} - \sum_{m=1}^{M}\sum_{s=1}^{3}\frac{1}{1-\epsilon_{(\rm sp)}^{(s)}(m)}\right\} \tag{4.31}$$

where the relation between $\epsilon_{(\rm sp)}^{(s)}(m)$ and the eigenvalues of (4.27) is given by (4.28). In accordance with the restriction imposed on the domain of integration in equation (2.30),

the upper limit in (4.31), M, is obtained from the inequality $q^2 \leqslant q_c^2$, if one makes the substitution

$$q^2 \to \Pi^2 \to \frac{4}{\lambda^2}(m + \tfrac{1}{2}). \tag{4.32}$$

Hence, $M = H_{tr}/B - 1$. It follows from (4.28) and (4.29) that

$$\sum_{s=1}^{3} \frac{1}{1 - \epsilon_{(sp)}^{(s)}(m)} = \frac{H_{tr}}{B} \sum_{s=1}^{3} \frac{1}{\xi_{(sp)}^{(s)}(m)} = \frac{H_{tr}}{B} \mathrm{Tr}\, \mathbf{H}^{-1}(m). \tag{4.33}$$

Thus we get the final result

$$\frac{\delta\sigma(B)}{\sigma_0} = \frac{1}{2\pi\epsilon_F\tau} \left\{ \sum_{m=0}^{M} \frac{1}{A_m} - \frac{1}{D_0} - \frac{B_0 + D_1}{B_0 D_1 - C_1^2} - \sum_{m=1}^{M} \frac{D_{m+1}D_{m-1} + B_m(D_{m+1} + D_{m-1}) - (C_m^2 + C_{m+1}^2)}{D_{m+1}D_{m-1}B_m - (C_m^2 D_{m+1} + C_{m+1}^2 D_{m-1})} \right\}. \tag{4.34}$$

The expressions (4.31) and (4.34) suggest the values of the magnetic field are so that the quantities H_{tr}/B are integer numbers more than two. By treating experimental data, one should first calculate $\delta\sigma(B)$ at such discrete points and then extend the function between the points with the help of a smoothing procedure of some kind. The result of such manipulations presented in figure 8 clearly displays the possibility of weak antilocalization.

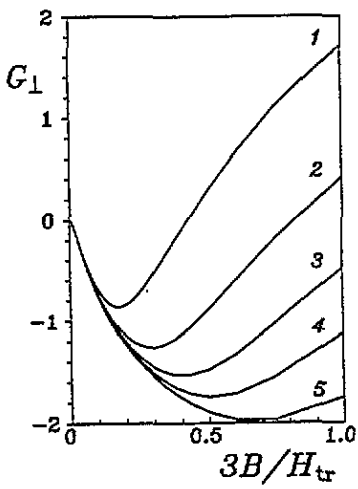

Figure 8. Theoretical results of the magnetoconductivity in a perpendicular field $B_\perp$ ($G_\perp = 2\pi\epsilon_F\tau(\sigma(B_\perp) - \sigma(0))/\sigma_0$) for the choice of H_ϕ=0.01(H_{tr}/3) for various values of $H_{so} = x(H_{tr}/3)$, x = 0.3 (1), 0.5 (2), 0.7 (3), 0.9 (4), and 1.2 (5). In drawing the function $G_\perp(B)$ between the discrete points $B_n = H_{tr}(n+3)^{-1}$, $n = 0, 1, 2, \ldots$ the linear extrapolation was used for the sake of simplicity.

5. Magnetoconductivity in a parallel field

A parallel magnetic field appears in the problem only through the Zeeman interaction. In this case, in order to solve the transport equation (2.14) the eigenfunctions of the kernel T of (2.17) are not necessary. One can immediately look for the solution in the form analogous to equation (3.19)

$$(\alpha\beta|K|\gamma\delta) = \frac{1}{2\pi\tau} \left\{ \left[A\delta_{\alpha\beta}\delta_{\gamma\delta} + B(\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\beta}(\mathbf{c}\cdot\boldsymbol{\sigma})_{\gamma\delta} + C_{ij}(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}^{i}(\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\delta}^{j} \right] + \left[M\cdot(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}(\mathbf{c}\cdot\boldsymbol{\sigma})_{\gamma\delta} + (\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\beta}N\cdot(\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\delta} \right] \right.$$

$$+\left[S(\mathbf{c}\cdot\boldsymbol{\sigma})_{\alpha\beta}\delta_{\gamma\delta}+U\delta_{\alpha\beta}(\mathbf{c}\cdot\boldsymbol{\sigma})_{\gamma\delta}\right]$$
$$+\left[\boldsymbol{L}\cdot(\mathbf{c}\times\boldsymbol{\sigma})_{\alpha\beta}\delta_{\gamma\delta}+\delta_{\alpha\beta}\boldsymbol{V}\cdot(\mathbf{c}\times\boldsymbol{\sigma})_{\gamma\delta}\right]\Big\}. \tag{5.1}$$

The substitution of (5.1) into (2.14) results in a set of equations from which the scalars A, B, S, and U, the 2D vectors $\boldsymbol{M}, \boldsymbol{N}, \boldsymbol{L}$, and $\boldsymbol{V}$, and the 2×2 matrix C_{ij} may be determined. These equations are

$$\begin{aligned}
A &= 1 + aA - \mathrm{i}\Omega\boldsymbol{L}\cdot(\mathbf{c}\times\boldsymbol{h})\\
B &= 1 + bB + \boldsymbol{M}\cdot\boldsymbol{s}\\
C_{ij} &= \delta_{ij} + dC_{ij} - s_iN_j - \mathrm{i}\Omega(\mathbf{c}\times\boldsymbol{h})_iV_j\\
M_i &= dM_i - \mathrm{i}\Omega U(\mathbf{c}\times\boldsymbol{h})_i - Bs_i\\
N_j &= bN_j + s_iC_{ij}\\
S &= bS + \boldsymbol{L}\cdot\boldsymbol{s}\\
U &= aU - \mathrm{i}\Omega\boldsymbol{M}\cdot(\mathbf{c}\times\boldsymbol{h})\\
L_i &= dL_i - \mathrm{i}\Omega A(\mathbf{c}\times\boldsymbol{h})_i - Ss_i\\
V_j &= aV_j - \mathrm{i}\Omega(\mathbf{c}\times\boldsymbol{h})_iC_{ij}
\end{aligned} \tag{5.2}$$

where

$$\boldsymbol{s} = t(\hat{\boldsymbol{q}}\times\boldsymbol{c}) \qquad t = \mathrm{i}\eta Q \tag{5.3}$$

and a, b, and d are the functions introduced in equation (3.3). It is seen that (5.2) splits into three systems of the same type for the sets of functions $(A, S, \boldsymbol{L})$, $(B, U, \boldsymbol{M})$, and $(\boldsymbol{N}, \boldsymbol{V}, C_{ij})$. Let us define a symmetric 2×2 matrix with elements D_{ij},

$$D_{ij} = (1-b)(1-d)\delta_{ij} + s_is_j + \Omega^2\left(\frac{1-b}{1-a}\right)(\mathbf{c}\times\boldsymbol{h})_i(\mathbf{c}\times\boldsymbol{h})_j. \tag{5.4}$$

Then the solution of the system (5.2), which can be obtained by standard linear algebra methods, has the form

$$A = \frac{1}{1-a}\left[1 - \Omega^2\left(\frac{1-b}{1-a}\right)(\mathbf{c}\times\boldsymbol{h})_iD_{ij}^{-1}(\mathbf{c}\times\boldsymbol{h})_j\right] \tag{5.5}$$

$$B = \frac{1}{1-b}(1 + \boldsymbol{M}\cdot\boldsymbol{s}) \tag{5.6}$$

$$C_{ij} = (1-b)D_{ij}^{-1} \tag{5.7}$$

$$M_i = -N_i = -D_{ik}^{-1}s_k \tag{5.8}$$

$$S = -U = -\frac{\mathrm{i}\Omega}{1-a}s_iD_{ij}^{-1}(\mathbf{c}\times\boldsymbol{h})_j \tag{5.9}$$

$$L_j = V_j = -\mathrm{i}\Omega\left(\frac{1-b}{1-a}\right)D_{jk}^{-1}(\mathbf{c}\times\boldsymbol{h})_k \tag{5.10}$$

where

$$D_{nm}^{-1} = \frac{1}{EG-F^2}\left\{G\hat{q}_n\hat{q}_m - F[\hat{q}_n(\hat{\mathbf{q}}\times\mathbf{c})_m + \hat{q}_m(\hat{\mathbf{q}}\times\mathbf{c})_n] + E(\hat{\mathbf{q}}\times\mathbf{c})_n(\hat{\mathbf{q}}\times\mathbf{c})_m\right\} \tag{5.11}$$

and we have introduced

$$\begin{aligned}
E &= (1-b)(1-d) + \cos^2\phi\,\underline{\Omega}^2\\
F &= -\cos\phi\sin\phi\,\underline{\Omega}^2\\
G &= (1-b)(1-d) + t^2 + \sin^2\phi\,\underline{\Omega}^2\\
\underline{\Omega}^2 &= \Omega^2\left(\frac{1-b}{1-a}\right)
\end{aligned} \tag{5.12}$$

and ϕ is the polar angle of the vector $\hat{q}$. Substituting (5.1) in (2.30) and carrying out the integration over ϕ, we get

$$\frac{\delta\sigma(B)}{\sigma_0} = \frac{1}{4\pi\epsilon_F\tau}\int_0^1 dz\left[\frac{\gamma(\gamma-2\eta^2 z)}{1-a} - (1-d)(\gamma+\underline{\Omega}^2) - (1-b)(2\gamma-2\eta^2 z+\underline{\Omega}^2)\right]$$
$$\times\left[\gamma(\gamma-2\eta^2 z)(\gamma+\underline{\Omega}^2)(\gamma-2\eta^2 z+\underline{\Omega}^2)\right]^{-1/2} \tag{5.13}$$

where

$$z = Q^2/2 \qquad \gamma = (1-b)(1-d). \tag{5.14}$$

From (5.13), it follows that in the low-magnetic-field limit

$$\frac{\delta\sigma(B)-\delta\sigma(0)}{\sigma_0} = \frac{-1}{2\pi\epsilon_F\tau}(\omega_s\sqrt{\tau_\epsilon\tau_{so}}) \tag{5.15}$$

at $\tau_{so} \ll \tau_\epsilon$, and

$$\frac{\delta\sigma(B)-\delta\sigma(0)}{\sigma_0} = \frac{-1}{96\pi\epsilon_F\tau}(\omega_s\tau_\epsilon)^2\left(\frac{\tau_\epsilon}{\tau_{so}}\right) \tag{5.16}$$

at $\tau_\epsilon \ll \tau_{so}$. The magnetoconductivity in a wider interval, $0 \leqslant \omega_s\tau_{so} \leqslant \frac{1}{3}$, obtained by means of the numerical integration of (5.13) is presented in figure 9.

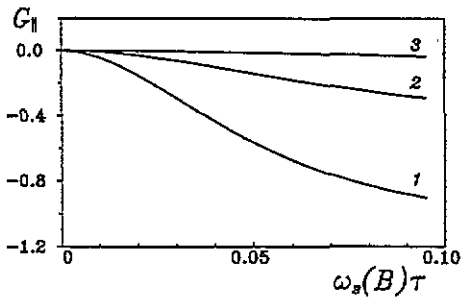

Figure 9. Theoretical results of the magnetoconductivity in a parallel field $B_\parallel$ ($G_\parallel = 4\pi\epsilon_F\tau(\sigma(B)-\sigma(0))/\sigma_0$) for the choice of τ/τ_{so}=0.3 for various values of τ/τ_ϵ: 0.01 (1), 0.03 (2), 0.1 (3).

6. Comments on systems with the $H_{so}^{(3.1)}$ Hamiltonian

In this section, we show that the magnetoconductivity of a system whose SO dynamics is controlled by the Hamiltonian $H_{so}^{(3.1)}$ of (1.3) coincides (at equal SO constants α and β) with that of a system governed by the Hamiltonian $H_{so}^{(1)}$ of (1.1). The proof is based on comparing the conductivity graphs of the same type for the two systems. First note that the Hamiltonians of the SO and Zeeman interactions, $H_{so}^{(3.1)}$ and H_Z, transform into

$$\tilde{H}_{so}^{(3.1)}(p) = \beta(p_x\sigma_y + p_y\sigma_x) \tag{6.1}$$

$$\tilde{H}_Z = \tfrac{1}{2}\omega_s b\cdot\sigma \qquad b = (b_x, b_y, b_z) = (h_y, -h_x, h_z) \tag{6.2}$$

under the rotation in the spin space about the $\hat{z}$ axis by an angle of $\pi/2$. This transformation, being a canonical one, does not affect the system dynamics. Since p_x and p_y are the integration variables, one can introduce new variables

$$(p_x, p_y) = (-\tilde{p}_x, \tilde{p}_y). \tag{6.3}$$

This change of variables transforms $\tilde{H}_{so}^{(3.1)}(p)$ into $\tilde{H}_{so}^{(3.1)}(\tilde{p})$

$$\tilde{H}_{so}^{(3.1)} = \beta(-\tilde{p}_x\sigma_y + \tilde{p}_y\sigma_x) = \beta\tilde{p}\cdot(\hat{z}\times\sigma) \tag{6.4}$$

which has the same form as $H_{so}^{(1)}(p)$. Now it is not difficult to understand that in the case of a parallel magnetic field the transformation (6.3), accompanied by replacing the unit vector b with h, makes an one-to-one correspondence between the Feynman graphs of the $\tilde{H}_{so}^{(3.1)}(\tilde{p})$ theory and those of the $H_{so}^{(1)}$ theory. Although the transformation (6.3) changes the sign of the x component of the velocity vertex, p/m, this fact has no effect on the diagonal components of the velocity correlation function. So the magnetoconductivities at the parallel field coincide.

In order to see the coincidence at perpendicular fields, the following arguments should be added. The part of the kernel $T^{(3.1)}$ of the transport equation for the $H_{so}^{(3.1)}$ system, which is linear in wave vector, $T_1^{(3.1)}(q)$, after the rotation in the spin space and the change of variables (6.3) acquires the form

$$\begin{aligned}(\alpha\beta|T_1^{(3.1)}|\gamma\delta) &= (\alpha\beta|T_1(\tilde{q})|\gamma\delta)\\ &= \frac{\mathrm{i}}{2}\eta l\left[(\tilde{q}_+\sigma_- + \tilde{q}_-\sigma_+)_{\alpha\beta}(\hat{z}\cdot\sigma)_{\gamma\delta} - (\hat{z}\cdot\sigma)_{\alpha\beta}(\tilde{q}_+\sigma_- + \tilde{q}_-\sigma_+)_{\gamma\delta}\right].\end{aligned} \tag{6.5}$$

Since

$$\tilde{q}_+ = \frac{1}{\sqrt{2}}(\tilde{q}_x + \mathrm{i}\tilde{q}_y) = -q_- \qquad \tilde{q}_- = \frac{1}{\sqrt{2}}(\tilde{q}_x - \mathrm{i}\tilde{q}_y) = -q_+ \tag{6.6}$$

it is seen that the matrix of the $T_1^{(3.1)}(q)$ operator in the f basis (4.5) is the transpose of the $T_1(q)$ matrix. The same is true for the operator $T_1^{(3.1)}(\mathbf{II})$ and, consequently, for the total kernel $T^{(3.1)}$. Therefore, the sum in equation (4.32), determined only by traces of the corresponding matrices, must be the same for the two systems. So magnetoconductivity measurements in the weak-localization regime cannot distinguish between $H_{so}^{(1)}$ and $H_{so}^{(3.1)}$.

7. Summary

In this work a theory is presented which describes the weak-localization correction to the conductivity in purely 2D semiconductor systems with the band SO interaction. A transport equation for the Cooperon propagator is derived which, because of the lack of central symmetry, is no longer reducible to the ordinary diffusion equation. It is established that the behaviour of the conductivity resembles that of metal films with impurity SO scattering in the sense that, depending on the strength of the SO interaction, the localization can transform into antilocalization and the magnetic-field dependence can be of variable sign. The explicit expression for the quantum correction is, however, different in the two theories. The results of the present work suggest that some recent measurements of the magnetoconductivity in 2D heterostructures should be re-examined in the light of the theory developed.

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Note added in proof. Since this paper was submitted for publication, a short note by Iordanskii *et al* has appeared [19], dealing with a similar problem. They studied 2D systems with the Hamiltonian $H_{so}^{3,1}$ under the perpendicular

magnetic field. Their results are different from ours. It is difficult to identify the exact reason for the disagreement because there were no derivations in that paper. We, however, do not rule out the possibility that the difference can be attributed to a difference between the approaches applied. In the present calculation, Green functions were used, which exactly took into account the so interaction. Instead of this, as far as we can ascertain, the approach of [19] is based on a hypothetical possibility of considering the band so interaction as the electromagnetic interaction with a fictitious spinor vector potential. We do not share this opinion that such an approach is tenable. Even if such a potential could exist, it would fall far ouside the Maxwell theory. There are, for example, good reasons for doubting whether the semiclassical phase-integral approximation (similar to equation (4.1), which is a direct consequence of the gauge invariance) would be correct in the case considered. Therefore, we believe that any results obtained within such an approach should be treated with great caution.

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